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RESEARCH MEMORANDUM

NONUNIFORM BURNUP AND POISONING EFFECTS IN A REACTOR
AND VALIDITY OF UNIFORM APPROXIMATION

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SUMMARY

The nuclear-reactor simulator at the NACA Lewis laboratory has been applied to an investigation of the validity of the customary assumption of uniform distributions of fuel burnup and fission-product poisoning in a reactor core. The two-group calculation results show the effects of this approximation on the reactivity, power distribution, and xenon-135 distribution for a specific power reactor in which the initial power density varied by a factor of approximately 4 over the core volume.

For this reactor, the nonuniform fuel burnup and poisoning were satisfactorily approximated by equivalent uniform distributions of burnup and poisoning in the reactor core for the conditions of 10-percent fuel burnup or less and for equilibrium xenon-135 concentrations. Only slight distortions of 4 percent or less in power density resulted from the approximations. However, for operation at the maximum xenon-135 concentration condition approximately 10 hours 50 minutes after shutdown, the approximate power density solutions diverged from the more detailed results by as much as 24 percent at the reactor center.

INTRODUCTION

The burnup of fuel and the consequent generation of fission poisons in a reactor are important considerations in the design of a power reactor, particularly a reactor designed for long life. Perturbation calculations based on the original clean reactor condition are accurate

for determining only small changes in reactivity. For large changes in fuel or poison concentrations, each reactor condition must therefore be treated in an independent reactivity calculation. In these independent calculations the burnup and poison effects are usually approximated by considering the fuel to be depleted uniformly throughout the reactor core and the poisons similarly uniformly distributed and dependent on some average thermal neutron flux for the reactor. In many cases, however, the question arises as to whether it is necessary to account for the actual spatial distribution of the fuel burnup and of the poisons in the determination of reactivity and of variation in power density.

In order to provide information on the validity of this usual approximation, calculations were carried out using both the detailed variations in fuel and poison concentrations and the approximate uniform values for a reactor with a considerable variation in power density over the core volume. The calculations were made for the following operating conditions:

- I. Hot, freshly loaded reactor.
- II. Hot reactor, equilibrium xenon, 10 megawatts power
- III. Hot reactor, equilibrium xenon, 100 megawatts power
- IV. Hot reactor, equilibrium xenon, 10-percent fuel burnup, 100 megawatts power
- V. Hot reactor, maximum xenon following shutdown for 10 hours 50 minutes, 10-percent fuel burnup.

The detailed values for local fuel and poison concentrations were used in calculations made for each of the listed conditions. The first three conditions establish the reactivity and poisoning characteristics of the system for operation at increasing power levels. For the last three of these conditions, additional calculations were made based on a uniform distribution of the total uranium-235 burnup and of the total fission poisons determined in the more exact calculations. These calculation results are then compared to show the effect of the approximation of uniform distribution on reactivity and power density values in the reactor core.

The calculations were made at the NACA Lewis laboratory and were facilitated by use of the nuclear reactor simulator in which both fuel and poison concentrations could be varied to satisfy the conditions giving their spatial distributions.

SYMBOLS

The following symbols are used in this report:

b	fractional burnup of fuel
k	neutron multiplication constant
L^2	neutron migration area or age
N	atomic concentration
P	power density
p	resonance escape probability
R	ratio $N_{i,eq}/N_{x,eq}$
t	time variable
y	fractional yield from fission process
λ	radioactive decay constant
λ_{tr}	transport free path for neutrons
ν	average number of neutrons per fuel atom fission
Σ	macroscopic neutron absorption cross section
Σ_f	macroscopic neutron fission cross section
σ	microscopic neutron absorption cross section
τ	reactor core volume
ϕ	neutron flux
ϕ_x^*	thermal neutron flux at which burn-up rate for Xe^{135} equals natural decay rate

Subscripts:

a	property of core materials other than fuel or poison
eq	equilibrium
f	fast neutron group

i	property of iodine-135
max	maximum
p	property of stable fission poisons
th	thermal neutron group
u	property of uranium-235
x	property of xenon-135
0	value for newly assembled reactor core

ANALYSIS OF POISONING AND BURNUP

Poisoning by Xenon-135

The generation and destruction of Xe^{135} atoms in a reactor is described by the following equations for the atomic concentrations N_i for I^{135} and N_x for Xe^{135} (ref. 1). Other quantities are identified in the list of symbols.

$$\frac{dN_i}{dt} = y_i(\Sigma_{f,f} \phi_f + \Sigma_{f,th} \phi_{th}) - \lambda_i N_i \quad (1)$$

$$\frac{dN_x}{dt} = y_x(\Sigma_{f,f} \phi_f + \Sigma_{f,th} \phi_{th}) + \lambda_i N_i - N_x \sigma_x \phi_{th} - \lambda_x N_x \quad (2)$$

In these equations it is assumed that only neutrons of the thermal group produce an appreciable burnup of Xe^{135} atoms and that there is negligible burnup of I^{135} atoms by either fast or thermal neutrons.

Equilibrium Xe^{135} concentration. - When equilibrium is reached in the production and loss of Xe^{135} atoms,

$$dN_i/dt = dN_x/dt = 0$$

and

$$N_{i,eq} = (y_i/\lambda_i)(\Sigma_{f,f} \phi_f + \Sigma_{f,th} \phi_{th}) \quad (3)$$

$$N_{x,eq} = \frac{(y_i + y_x)(\Sigma_{f,f} \phi_f + \Sigma_{f,th} \phi_{th})}{\lambda_x + \sigma_x \phi_{th}} \quad (4)$$

The ratio of the equilibrium concentrations of I^{135} and Xe^{135} is then

$$R = \frac{N_{I,eq}}{N_{X,eq}} = \frac{\lambda_X + \sigma_X \phi_{th}}{\lambda_I (1 + y_X/y_I)} \quad (5)$$

and the macroscopic absorption cross section for Xe^{135} ($\Sigma_X = N_X \sigma_X$) is

$$\Sigma_X = \frac{(y_I + y_X)}{(\phi_X^* + \phi_{th})} (\Sigma_{f,f} \phi_f + \Sigma_{f,th} \phi_{th}) \quad (6)$$

where

$$\phi_X^* = \lambda_X / \sigma_X \quad (7)$$

Xenon build-up after shutdown. - The condition of reactor shutdown after operation with equilibrium Xe^{135} and I^{135} concentrations leads to the following solutions for equations (1) and (2) with $t = 0$ at the time of reactor shutdown.

$$N_I = N_{I,eq} e^{-\lambda_I t} \quad (8)$$

$$N_X = N_{X,eq} e^{-\lambda_X t} + \frac{\lambda_I N_{I,eq}}{\lambda_I - \lambda_X} (e^{-\lambda_X t} - e^{-\lambda_I t}) \quad (9)$$

These equations give a Xe^{135} absorption cross section relative to the equilibrium value $\Sigma_{X,eq}$ of

$$\frac{\Sigma_X}{\Sigma_{X,eq}} = e^{-\lambda_X t} + \frac{R \lambda_I}{\lambda_I - \lambda_X} (e^{-\lambda_X t} - e^{-\lambda_I t}) \quad (10)$$

and a maximum for the Xe^{135} absorption cross section at the time

$$t_{max} = \frac{1}{\lambda_I - \lambda_X} \ln \left(\frac{R \lambda_I^2}{\lambda_X (\lambda_I - \lambda_X) + R \lambda_I} \right) \quad (11)$$

The dependence of t_{max} on R , and therefore on ϕ_{th} , gives an interval over which a maximum Xe^{135} concentration appears in various regions in a reactor. This variation is apparent in figure 1 where the time for maximum Xe^{135} concentration is plotted as a function of the thermal neutron flux in a reactor having thermal neutrons in equilibrium with a temperature of $1350^\circ F$. Sections of a typical reactor having a core flux variation of 1 to 2×10^{14} neutrons per square centimeter per second reach maximum Xe^{135} concentrations over a period of 45 minutes.

Fuel Burnup and Effect of Stable Poisons

Assumptions. - The effects of the burnup of fuel and the generation of fission-product poisons other than Xe^{135} on reactivity and flux calculations can be treated by a simplified analysis based on the following assumptions:

(1) The average microscopic fission, absorption, and scattering cross sections for the enriched nuclear fuel are not changed by the selective burnup of U^{235} atoms.

(2) Only the resonance escape probability p_{th} and the multiplication constant k_f for the fast neutron group will vary with fuel burnup. The diffusion and slowing properties of the reactor core remain unaffected by burnup or by poison generation.

(3) The variation with time of the fission-rate distribution in the reactor is small for small amounts of burnup (10 percent); therefore, integrated burnup and poisoning effects can be based on the initial distribution. (This assumption is evaluated in the section describing the results of calculations.)

(4) The poisons other than Xe^{135} are assumed to be stable or to have decay periods long in comparison with the lifetime of the reactor fuel elements. They are further considered to have cross sections small enough that they are burned up at a negligible rate by either the fast or the thermal neutron flux.

(5) The effect of these other poisons will be felt only through the absorption of thermal neutrons. This absorption is treated by the specification of an average thermal absorption cross section per fuel atom destroyed.

Calculation of thermal group parameters. - When a fraction b of the nuclear fuel in a reactor has been burned up, the macroscopic thermal absorption cross section for the fuel can be related to its initial value $\Sigma_{u,th,0}$

$$\Sigma_{u,th} = (1 - b)\Sigma_{u,th,0} \quad (12)$$

and the thermal absorption cross section for the stable poisons generated in the process is

$$\Sigma_{p,th} = b \Sigma_{u,th,0} \left(\frac{\sigma_p}{\sigma_u} \right) \quad (13)$$

The burnup fraction at any point in the reactor core after a time t is defined as

$$b = \frac{N_{u,0} - N_u}{N_{u,0}} \quad (14)$$

and can be calculated from the rate of destruction of fuel atoms by

$$b = \frac{1}{N_{u,0}} \int_0^t (\Sigma_{u,f} \phi_f + \Sigma_{u,th} \phi_{th}) dt$$

If the average burnup fraction \bar{b} for the reactor core of volume τ is

$$\bar{b} = \frac{\int_0^t \int_{\tau} (\Sigma_{u,f} \phi_f + \Sigma_{u,th} \phi_{th}) dt d\tau}{\int_{\tau} N_{u,0} d\tau}$$

then

$$\bar{b} = \frac{\int_0^t (\Sigma_{u,f} \phi_f + \Sigma_{u,th} \phi_{th}) dt \int_{\tau} N_{u,0} d\tau}{\int_0^t \int_{\tau} (\Sigma_{u,f} \phi_f + \Sigma_{u,th} \phi_{th}) dt d\tau N_{u,0}} \quad (15)$$

With the use of the approximation of constant fission rate distribution with time, equation (15) reduces to

$$\bar{b} = \frac{\Sigma_{u,f} \phi_f + \Sigma_{u,th} \phi_{th}}{\int_{\tau} (\Sigma_{u,f} \phi_f + \Sigma_{u,th} \phi_{th}) d\tau} \frac{\int_{\tau} N_{u,0} d\tau}{N_{u,0}} \quad (16)$$

For thermal neutrons, the absorption cross section for fuel atoms is replaced by a composite cross section for fuel and poison atoms

$$\Sigma_{p,th} + \Sigma_{u,th} = \left(1 - b + b \frac{\sigma_p}{\sigma_u}\right) \Sigma_{u,th,0} \quad (17)$$

whereas the thermal fission cross section becomes

$$\Sigma_{f,th} = (1 - b) \Sigma_{f,th,0} \quad (18)$$

Equations (17) and (18) then give a thermal multiplication constant

$$k_{th} = \frac{\nu \Sigma_{f,th}}{\Sigma_{a,th} + \Sigma_{p,th} + \Sigma_{u,th}} = \frac{\nu(1 - b) \Sigma_{f,th,0}}{(\Sigma_{a,th} + \Sigma_{u,th,0}) - b \left(1 - \frac{\sigma_p}{\sigma_u}\right) \Sigma_{u,th,0}}$$

or

$$k_{th} = \frac{1 - b}{\frac{1}{k_{th,0}} - b \left(1 - \frac{\sigma_p}{\sigma_u}\right) \frac{\Sigma_{u,th,0}}{\nu \Sigma_{f,th,0}}} \quad (19)$$

Calculation of fast group parameters. - For the fast neutron group, the absorption due to poisons is neglected as stated in the assumptions. Therefore, it is necessary to determine only the variations in the quantities p_{th} and k_f with the depletion in fuel concentration. For a given reactor configuration, these values are generally calculated in advance as functions of fuel concentration and are applied to local core volume elements where b/\bar{b} has been determined by equation (16).

REACTIVITY CALCULATIONS

Reactor Constants

The reactor used for sample calculations of the fuel burnup and poisoning effects consisted of a beryllium carbide (Be_2C) moderated, liquid-lead-cooled core with U^{235} as fuel and a Be and Pb reflector. The core was a right cylinder, 2 feet in diameter and 2 feet high. The double reflector consisted of 1/4 inch of Be and 4 inches of Pb. The reactor operated with thermal neutrons in equilibrium with the Be_2C at an average temperature of 1350°F . The composition of the core is tabulated in table I and the appropriate fast and thermal neutron constants for the newly assembled core are listed in table II.

The core radius and properties of the reflector materials gave a considerable variation in both fast and thermal fluxes with position in the core. This was due to the almost negligible moderating properties of the reflector structure, although the neutron reflecting properties of this region kept the power density values shown in figure 2 from falling off by a factor as large as 4 in the core. This power variation intensified the distribution effect of the poison and burnup processes.

Reactor Simulator Calculations

The calculations of the effects of poisons and of fuel burnup on reactor fluxes and reactivity were performed on the NACA nuclear reactor simulator (ref. 2). Standard two-group calculation approximations, with varying local neutron parameters determined according to equations previously shown in the analysis, led to reactivity and flux values for the reactor under specified operating conditions. The nuclear poison constants used in the calculations appear in table III.

The resistance network of the simulator consisted of 13 sections representing core volume elements and one section giving core-reflector boundary conditions (ref. 3). For a given neutron group in the reactor core, all processes such as slowing, axial leakage, absorption in all materials including uranium, and poisons other than Xe^{135} were simulated by a single resistor for each volume element. A separate resistor in each thermal group network section simulated the absorption by Xe^{135} .

A typical calculation for a poisoned reactor proceeds according to the following program based on a previous calculation for the unpoisoned condition:

(1) Network resistors are changed to include the effect of any fuel burnup and stable poisons given by equations (16) and (17).

(2) Trial Xe^{135} resistor values are calculated by equation (6) with estimated fluxes and fission distributions or values from the previous calculation.

(3) The reactor calculation is repeated, including any burnup and stable poisons and the trial Xe^{135} distribution, to give an approximate solution for the fluxes and power density.

(4) Steps (2) and (3) are repeated by an iteration process until consistent values for fluxes and Xe^{135} concentrations result.

The process generally converges rapidly on the required reactor solution giving fluxes, power density, and poison distributions for the noncritical reactor. Comparisons based on these noncritical solutions permit use of one reactor composition for all operating conditions; the primary effects shown are then those of burnup and poisoning undistorted by control rods or additional fuel for Xe^{135} override. The reactivity is given by the fractional increase in power for one neutron-generation period, which corresponds to a single iteration of the calculation process.

RESULTS OF CALCULATIONS

The reactor operating conditions specified for the calculations and resulting reactivity values are listed in tables IV and V. The nonuniformly distributed poison calculations required an average of three to four iterations to give local Xe^{135} concentrations and fluxes consistent to within 1 percent. Therefore, no special methods were used to increase the convergence rate.

The reactor characteristics with regard to poisoning are shown by examples I, II, and III. The major reactivity change from 1.14 to 1.10 occurs with a power increase from 0 to 10 megawatts. A further reactivity change to 1.08 results with an increase to 100 megawatts. The reactor is therefore operating with nearly a maximum equilibrium Xe^{135} concentration.

The effect of an assumed uniform distribution of Xe^{135} is shown in calculation III, wherein the total Xe^{135} content determined in III is redistributed uniformly throughout the core. The uniform Xe^{135} concentration is approximated within 3 percent by equation (6) with the fission rate and thermal flux values averaged over the unpoisoned reactor core. The reactivity value of 1.08 for calculation III agrees well with the more exact calculation result and indicates that the approximation of uniform poison distribution is justified for this case.

The effect of uranium burnup in case IV is based on the use of the fission rate distribution of calculation III as an approximate average for the burnup period. That this is a reasonable approximation is apparent from figure 2, which shows the power density distributions for case III and for the 10-percent burnup, poisoned case IV. The two distributions differ by a maximum of approximately 3 percent. The total effect of burnup and xenon poisoning in the reactor is to reduce the reactivity to 1.06.

Here again the approximations of uniform fuel burnup and poison distributions can be used to give results comparable to those using the calculated distributions. Calculation IV', with these approximations, gives a reactivity of 1.07 in contrast to the value for case IV. Greater differences occur in the power density distributions for these two cases as shown in figure 3. The deviations of approximately 4 percent or less are not excessive for the single calculation but indicate progressively larger deviations with successive burnup periods in a long-lived reactor.

The errors introduced by the approximations of uniform burnup and poisoning reach their greatest values for the condition of maximum Xe^{135} concentration as shown by calculations V and V'. This condition occurs approximately 10 hours 50 minutes following a complete shutdown from operating condition IV. After this period of time all regions of the reactor core have maximum or nearly maximum concentrations of Xe^{135} and the reactivity has been reduced to approximately 0.698. The same amount of Xe^{135} together with burnup and stable poisons distributed uniformly in the reactor core leads to a reactivity value of 0.731 in calculation V'. This effect of distribution is not the only error to consider, for the conventional uniform poison calculation based on average fluxes and power densities fails to calculate the total Xe^{135} content of the core by approximately 6 percent. In addition to the reactivity disagreement, calculation V' gives power density values that differ from V by as much as plus 24 percent at the center of the reactor and minus 17 percent at the core-reflector interface. These power densities are not steady-state values. However, a reactor with sufficient fuel to override the maximum Xe^{135} effect would be expected to give corresponding large deviations in power density for the two solutions.

For large amounts of fuel burnup and fission poisons, therefore it is important to account for the effects of their distribution in the reactor core. The usual approximation of uniform burnup and poisoning is restricted to conditions giving only small variations of these processes with position. In addition, for all operating conditions considered, this approximation gave power densities at the center of the reactor that overestimated the actual values. For reactors with outputs limited by local power density values, the approximate solutions would give pessimistic predictions of available total power output.

CONCLUSIONS

Nuclear reactor calculations involving fuel burnup and poison generation were investigated with the NACA nuclear reactor simulator. On the average only three to four iterations were required to give

criticality, flux, and nonuniform poison distribution solutions within the limits of error of the system. With these detailed calculations as criteria, additional calculations using the customary approximations of uniform burnup and poison generation were made for comparison. Results of the comparison show:

1. For a reactor operating at 100 megawatts power with only equilibrium xenon poisoning, the solutions agreed within 1 percent for both reactivity and flux values.
2. For the reactor after 10-percent fuel burnup, the reactivity values differed by approximately 1 percent and the power densities by 4 percent or less.
3. The maximum xenon condition 10 hours and 50 minutes following instantaneous shutdown from the previous condition gave reactivity values differing by approximately 3 percent and local power density values differing by approximately 24 percent or less.
4. All solutions based on the uniform distribution approximations overestimated the values of maximum power density, which occurred at the center of the reactor.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, March 9, 1953

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TABLE I. - REACTOR CORE COMPOSITION

Component	Percent core volume	Number of atoms per cm^3
Pb (1300° F)	17.4	0.005099×10^{24}
Be_2C (1350° F)	82.3	.03745
U235	~.3	.0000724

TABLE II. - TWO-GROUP CONSTANTS FOR NEWLY ASSEMBLED REACTOR CORE

Transport free path for fast neutrons, $\lambda_{tr,f}$, cm	1.97
Fast neutron migration area, L_f^2 , cm^2	102
Fast neutron multiplication constant, k_f	1.807
Thermal neutron resonance escape probability, p_{th}616
Transport free path for thermal neutrons, $\lambda_{tr,th}$, cm	1.69
Macroscopic thermal neutron absorption cross section for U235 and other core materials exclusive of poison, $\Sigma_{a,th} + \Sigma_{u,th}$, cm^{-1}	0.02108
Thermal neutron multiplication constant for newly assembled reactor, $k_{th,0}$	1.994

TABLE III. - TENTATIVE POISON CONSTANTS

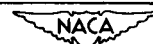
Constant	Value	Reference
Fission yield for I135, y_i	0.060	4
Fission yield for Xe135, y_x	.003	4
Decay constant for I135, λ_i , sec^{-1}	2.86×10^{-5}	4
Decay constant for Xe135, λ_x , sec^{-1}	2.1×10^{-5}	4
Stable poison cross section per U235 atom destroyed, σ_p (value for temperature of Hanford reactor)	75b	5
Xe135 absorption cross section σ_x (thermal neutrons 1350° F)	$1.55 \times 10^6 \text{b}$	6

TABLE IV. - REACTOR CONDITIONS AND REACTIVITY

Reactor condition	Power, megawatts	Reactivity
I Unpoisoned reactor	0	1.14
II Equilibrium Xe135 (actual distribution)	10	1.10
III Equilibrium Xe135 (actual distribution)	100	1.08

TABLE V. - COMPARISON OF REACTIVITIES FOR ACTUAL AND APPROXIMATE
POISON AND BURNUP DISTRIBUTIONS

Reactor condition		Power, megawatts	Reactivity	
			Actual	Approximate
Equilibrium Xe135	III	100	1.08	1.08
	III'	100		
10-percent U235 burnup; equilibrium Xe135	IV	100	1.06	1.07
	IV'	100		
10-percent U235 burnup; maximum Xe135 10 hr 50 min following shutdown	V	---	0.698	0.731
	V'	---		



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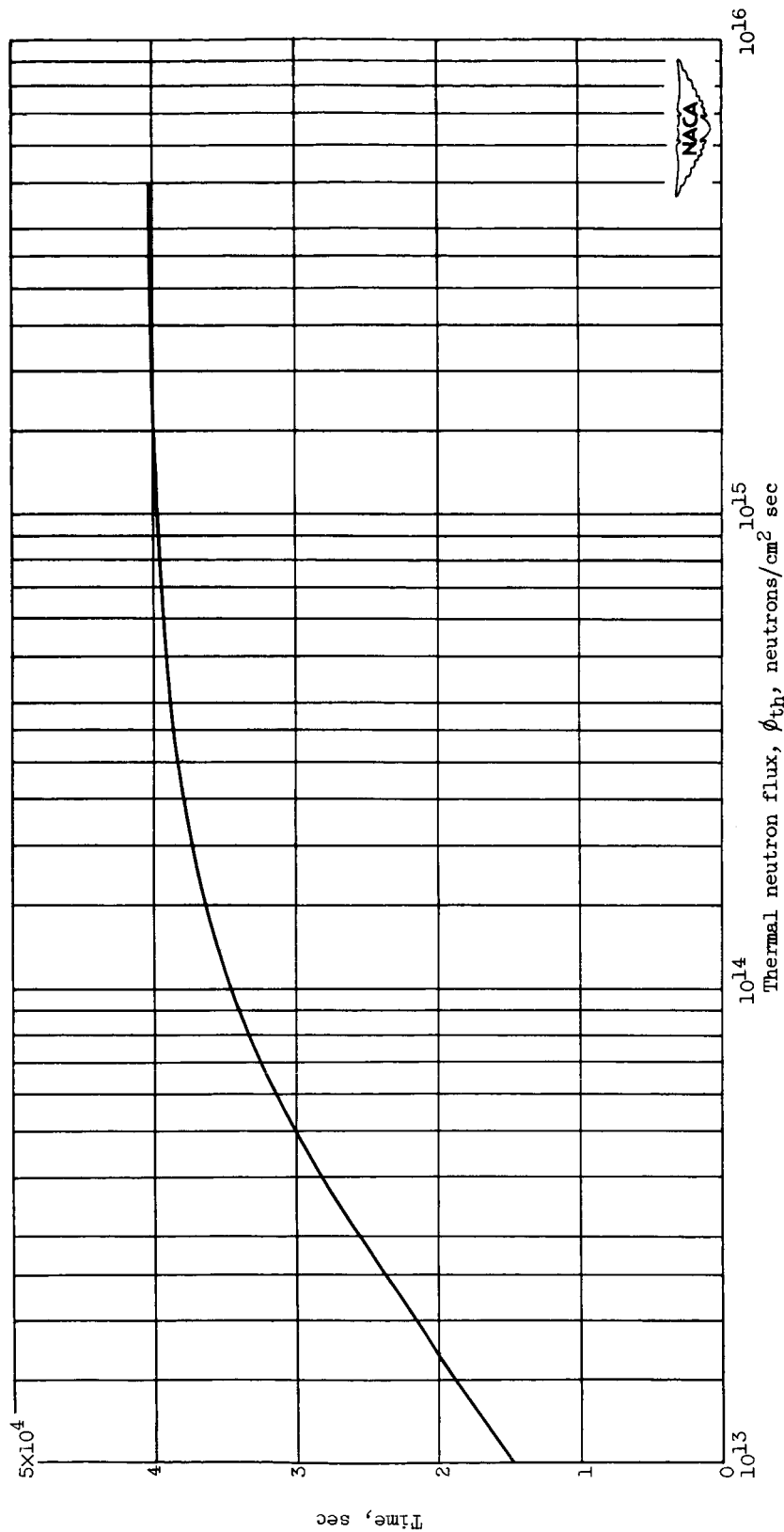


Figure 1. - Time for maximum xenon-135 concentration after shutdown as a function of equilibrium thermal neutron flux preceding shutdown.

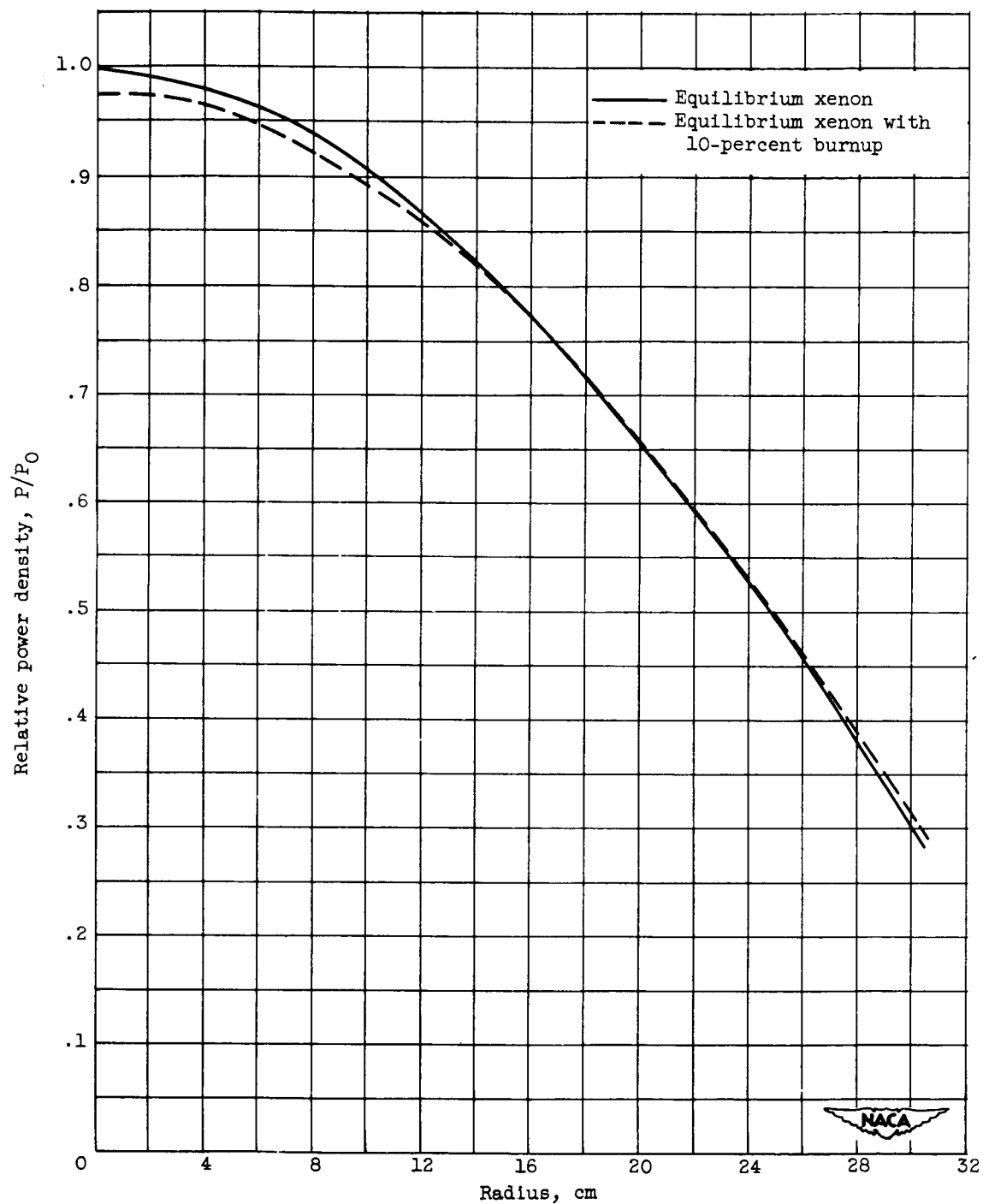


Figure 2. - Radial variation in relative power density for equilibrium xenon condition and for equilibrium xenon with 10-percent burnup. Power level, 100 megawatts.

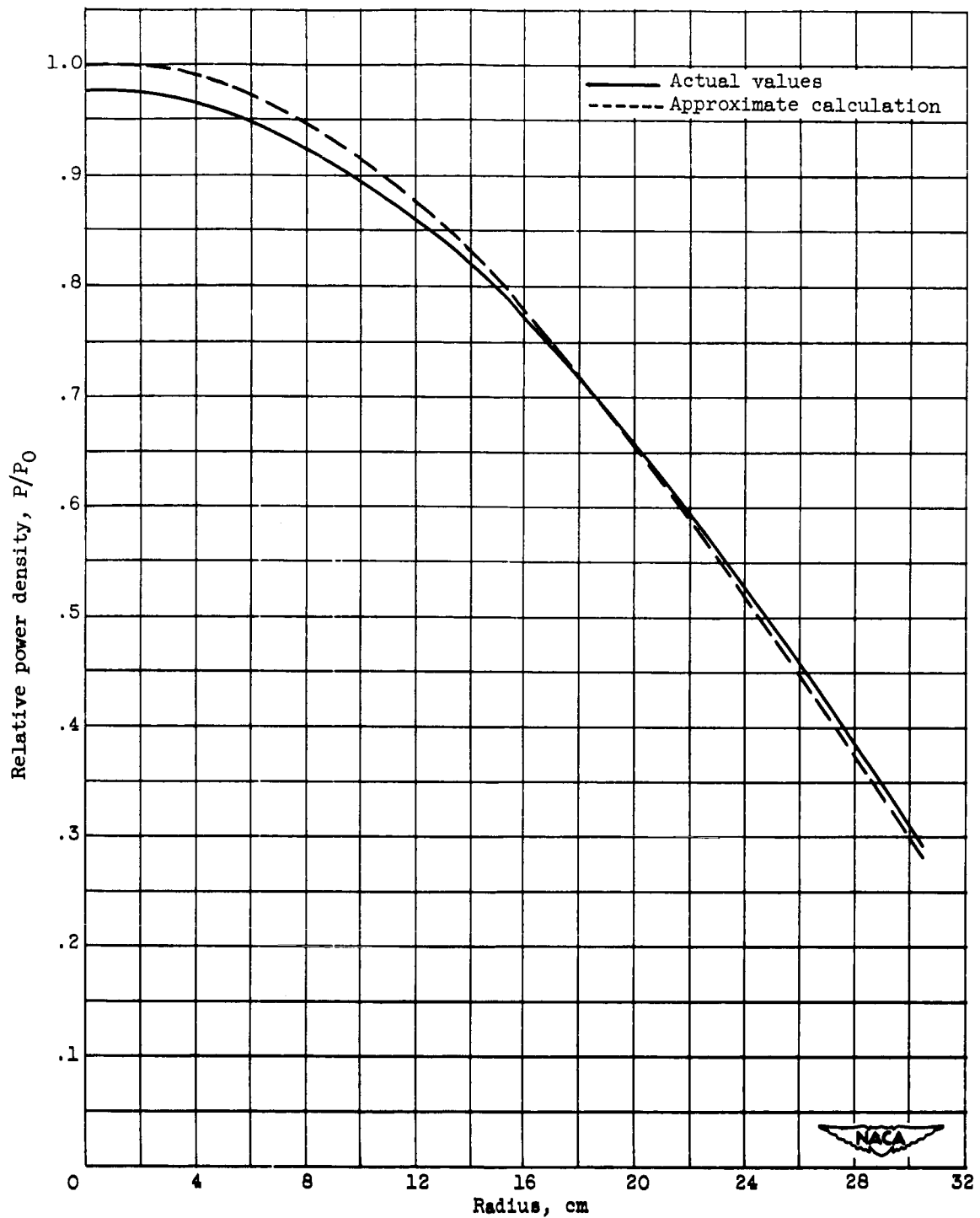


Figure 3. - Radial variation in relative power density determined by uniform approximation compared with results of detailed calculation. Power level, 100 megawatts; equilibrium xenon; 10-percent burnup.